Assessing Japanese Monetary Policy through Structural Bayesian VAR with time-varying parameters

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Abstract

After Nakajima, Shiratsuka, Teranishi, et al. (2010); Nakajima, Kasuya, and Watanabe (2011); Nakajima (2011) and among others have studied about how time-varying parameter Bayesian VAR can capture the movement of zero bound interest rate and is able to model the behavior of monetary policies in Japan. In our study we aim to capture the shocks by setting both short-term and long-term interest rate to very close to zero. By treating this crucial monetary policy from Bank of Japan as "unconventional monetary policy", we apply TVP-Structural BVAR with sign restriction in structural impulse response matrix. The results show that for industrial production growth rate, this variable is responsive to short-term interest rate relative to long-term (BOJ's discount rates). Despite that by using structural historical shock decompositions we found that during the past decades, the shocks from BOJ's discount rate has steadily affect to Japanese inflation.

Keywords— Bayesian Econometrics, State Space, Kalman Filter, Monetary Policy.

1 Introduction

During the past decades, Bayesian inferences are extensively developed to be compatible in econometric field. One is worth mentioning is Vector Autoregressive Regression. For example, Dieppe, Legrand, and Van Roye (2016) create Bayesian Econometric toolbox named as "BEAR Toolbox" with the effort to deliver the multiple types of Bayesian VAR packages. From the econometric point of view, we rather treat every observation to be random and dynamically

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change over time. With this being said, it is more logical for us to include the model that is capable of tracking variations for each specific in time.

For example, Primiceri (2005) develop Markov Chain Monte Carlo in Time-varying structural VAR and found that there was a statistical evidence proving that both systematic and non-systematic monetary policy have changed during the last 40 years in US. In addition high inflation and unemployment rate fluctuation are explained by non-policy shocks from monetary policy. To put into a simply word behind the concept of Primiceri (2005) is that they use algorithm from Carter and Kohn (1994) by allowing the interested state vector to be evolved with first-order random walk process via state space model.

Koop and Korobilis (2013) apply algorithm from Dynamic model averaging from Raftery, Kárný, and Ettler (2010) to estimate the time-varying parameter in VAR and found typical TVP-BVAR can be extended to include variables up to 135 variables. According to that algorithm, there is no need for MCMC and thus reduce tremendous of computational burden.

Despite above, it seems very promising that the TVP-VAR would and could capture the dynamic of macroeconomic observations. In practical world, however, econometricians improve to further theory by allowing the heterogeneity to be corresponding in TVP-VAR via the law of motions. This is so-called "Stochastic Volatility" where the variance in the disturbance term evolves via logarithm form. For instance Nakajima, Kasuya, and Watanabe (2011) develops the MCMC algorithm for TVP-VAR with stochastic volatility form Nakajima, Shiratsuka, Teranishi, et al. (2010) to evaluate the structural volatility change in four Japanese macroeconomic variables (Inflation, Industrial Production, nominal short-term interest rate and money supply). Miyao (2002) uncovers characteristics of variation in Japanese macroeconomic (interest rates, money supply, stock prices and output) from business fluctuations. Yano and Yoshino (2008) implement particle filtering to assess Japanese monetary policy reaction function. They found that the shocks from monetary policy actually has persistent effect on real output especially during the Japan "Bubble Economy" in late 1980s.

In this empirical work we apply TVP-BVAR based closely on Primiceri (2005) to estimate the impulse response and to investigate if what Bank of Japan has been done and been doing since the fight of Japan's deflation since 1990s is rational to their objectives. For more than decade that Japanese government has been dealing against deflation where gross domestic product has not been higher than 2%. Hence our primary goal is to study if monetary policy from Bank of Japan (BOJ) such as promising to set log-term bond yield (10 years) equals to zero helping to fight with deflation.

The article is organized as follow: In section 2 we introduce the specifications of TVP-BVAR model. Section 3 is Empirical Result and Section 4 is conclusion and policy implication.

2 Methodologies

The implementation of our TVP-BVAR relies in both Kalman filtering and by allowing state vector to be time variant. MCMC algorithm from Carter and Kohn (1994) for state-space model is applied. The baseline model is written as follow:

$$Y_t = X_t\beta_t + v_t \tag{1}$$

$$\beta_{t+1} = \mu + F\beta_t + e_t \tag{2}$$

$$VAR(v_t) = R \text{ and } VAR(e_t) = Q$$

where Y_t is $T \times N$ matrix containing the selected macroeconomic variables, X_t is a $T \times k$ matrix containing the regressor with parameter evolves over time via the state-space model in eq. (2). Random walk without drift is assumed in this series. Our model have no laws of motion. Therefore variance in disturbance term in eqs. (1) and (2) are constant overtime ie. $v_t \sim N(0, R)$ and $e_t \sim N(0, Q)$ where R and Q is constant over period. As in eq. (1), we treat β_t as unknown estimates. In order to obtain those estimates, Gibbs-sampling algorithm is applied with conditional posterior distribution. In this method we follow Kim, Nelson, et al. (1999) in chapter 8 closely in this description. The compact version of conditional distribution of state variable is given by the following:

$$P(\widetilde{\beta}_T|\widetilde{Y}_T) = P(\beta_T|\widetilde{Y}_T)\prod_{t=1}^{T-1} P(\beta_t|B_{t+1}, \widetilde{Y}_t) \tag{3}$$

One crucial assumption that is assumed is the disturbances of both observation equation ie. eq. (1) and Transition equation eq. (2) are mutually independent normally distributed. Our estimates conditional distribution collapses into:

$$\begin{aligned} P(\beta_T|\widetilde{Y}_T) &\sim N(\beta_{T|T}, P_{T|T}) \\ P(\beta_t|B_{t+1}, \widetilde{Y}_t) &\sim N(\beta_{t|t,\beta_{t+1}}, P_{t|t,\beta_{t+1}}) \end{aligned} \tag{4}$$

The interpretation of $\beta_{i|j}$ denotes an estimate of β at time i given information in Kalman filter upto time j. In order to finish the algorith we need to estimate mean and variance of the estimate above. This is when Kalman Filtering is taken part of.

The Kalman filtering include all recursive equation below.

$$\begin{aligned} \beta_{t|t-1} &= \mu + F\beta_{t-1|t-1} \\ P_{t|t-1} &= FP_{t-1|t-1}F' + Q \\ \eta_{t|t-1} &= Y_t - H\beta_{t|t-1} - Az_t \\ f_{t|t-1} &= HP_{t|t-1}H' + R \\ \beta_{t|t} &= \beta_{t|t-1} + P_{t|t-1}H'f^{-1}_{t|t-1}\eta_{t|t-1} \\ P_{t|t} &= P_{t|t-1} - P_{t|t-1}H'f^{-1}_{t|t-1}HP_{t|t-1} \end{aligned} \tag{5}$$

By finish running equations above from time $t = 1, 2, ...T$, at the end of resursive algorithm derives $\beta_{T|T}$ and $P_{T|T}$. By using backward recursive algorithm for state-space model from Carter and Kohn (1994), we obtain time-varying parameters in eq. (1).

2.1 Structural Impulse Response

A general VAR model with n endogenous variables, p lags and m exogenous predictors from eq. (1) can be written in another form as:

$$y_t = A_1 y_{t-1} + A_2 y_{t-2} + ... + A_p y_{t-p} + \epsilon_t \tag{6}$$

where $t = 1, 2, ..., T$ $y_t = (y_{1,t}, y_{2,t}, ..., y_{n,t})$ is a n x 1 vector of endogenous data, $A_1, A_2, ..., A_p$ are p matrices of coefficient for lag $1 - p$, respectively with dimension of $n \times n$. $\epsilon_t = (\epsilon_{1,t}, \epsilon_{2,t}, ..., \epsilon_{n,t})$ is a vector of residuals with mutually independent normal distribution assumption satisfied.

One can stack samples in the general way to estimate the model in a whole data set as follow:

$$Y = XB + \varepsilon \tag{7}$$

Alternatively for further computational advantage we can rewrite eq. (6), researcher can vectorise eq. (7) to reformulate the model as:

$$y = \overline{X}\beta + \epsilon \tag{8}$$

where

$$y = vec(Y), \overline{X} = I_n \otimes X, \beta = vec(B), \epsilon = vec(\varepsilon) \tag{9}$$

$$\epsilon \sim N(0, \overline{\Sigma}), where \overline{\Sigma} = \Sigma \otimes I_T \tag{10}$$

2.2 Historical Shock Decomposition

Consider eq. (6) By using backward substitution, one obtains

$$y_t = A_1 y_{t-1} + \epsilon_t = A_1(A_1 y_{t-2} + \epsilon_{t-1}) + \epsilon_t = A_1^2 y_{t-2} + \epsilon_t + A_1 \epsilon_{t-1} \tag{11}$$

Going on to more step further it can be written in a summarization form as:

$$y_t = \sum_{j=1}^{p} A_j^{(t)} y_{1-j} + \sum_{j=1}^{t-1} B_j \epsilon_{t-j} \tag{12}$$

It is worth to note that matrices $B_1, B_2, ..., B_{t-1}$ is corresponding to the response of y_t to shocks occurring at periods $t, t-1, ..., 2, 1$ and thus will be interpreted as impulse response function

$$B_j = \Psi^j \tag{13}$$

where

$$\Psi_j \epsilon_{t-j} = \Psi_j D D^{-1} \epsilon_{t-1} = \widetilde{\Psi}_j \eta_{t-j} \tag{14}$$

where DD^{-1} is cholesky decomposition matrix. Precisely, eq. (12) is separated into two parts: first is the shocks from exogenous regressor or lagged dependent variables and constant term and second is from unexpected structural shocks from error terms.

Now let's consider each specific variable i of the model where $i = 1, 2, ..., n$, one can put subscript i into the model as follow:

$$y_{i,t} = d_i^{(t)} + \sum_{j=0}^{t-1} (\widetilde{\phi}_{j,i1} \eta_{1,t-j} + \widetilde{\phi}_{j,i2} \eta_{2,t-j} + ... + \widetilde{\phi}_{j,in} \eta_{n,t-j}) \tag{15}$$

where $\widetilde{\phi}_{j,ik}$ represents (i, k) elements from structural impulse response matrix

2.3 Algorithm to compute historical decomposition

1. At iteration of i in Gibbs-sampling procedure, draw state vector (coefficients of SVAR in eq. (8) $\beta_{t,(i)}$)[2] and variance of both error term in observation equation ie. $\overline{\Sigma}_{(i)}$.

2. At iteration i From $t = 1, 2, ..., T$, compute the impulse response function matrices $\widetilde{\phi}_{t,(i)}$ from $\beta_{t,(i)}, \overline{\Sigma}_{(i)}$ and $D_{(i)}$.

3. At iteration i From $t = 1, 2, .., T$, derive residuals in eq. (8) $\epsilon_{t,(i)}$ by using $\beta_{t,(i)}$. After that obtain structural disturbance $\eta_{t,(i)}$. With $\eta_{t,(i)}$ and structural impulse response function $\widetilde{\phi}_{t,(i)}$. We are able to derive historical shock as in eq. (15).

3 Data Configurations

We consider a TVP-SVAR with two lags. Data here is Japanese macroeconomic variables, The time period is between 1957Q3 through 2017Q4. Our main objective is to find how output growth and inflation response statistically to monetary policies both in the past and in the future. By agree to set long-term government bond coupon to exact zero, we treat this policy as "Unconventional Monetary Policy". With this being said, it is thus logical for us to use impulse response function from TVP-SVAR to deliver the results. Given the time-varying parameters in TVP-BVAR we are able to capture if responding Japanese macroeconomic variable has changed over selected period. Our data set is transformed to be approximate stationary. Firstly output is percent change. Secondly Consumer Price Index is at level of Quarter on Quarter percent growth rate. Thirdly we expect treasury bill rate of Japanese government

[2] Note that $\beta_{t,(i)}$ denotes the coefficients drawn at i iteration at time t due to time-varying properties in state-space model.

bond as representative for "Short-Term Interest Rates". Finally is one of the main monetary policy tool from Bank of Japan "Discount Rates". The final variable is expected to represent the "Long-term Interest Rates". inally central bank's tool (Lending Rate) is at

4 Empirical Results

The Bank of Japan limits long term interest rate at zero percent as the objective of boosting overall Japanese economy and be able to tolerate against deflation. Theoretically speaking, it is thus rationale for us to assume that the response of industrial production growth, inflation to shocks of monetary policy[3] is positive as presented in tables 1 and 2.

In our empirical work, time-varying impulse response is provided in time series from TVP-VAR model. As time-varying in parameters of observation equation ie. eq. (7). Impulse response matrices are obtained for each period of time and calculated for each iteration of current draw of parameters from after posterior mean and variance is obtained.

According to figs. 3a and 3b it is statistically proved that industrial production index growth rate is highly sensitive to "Short-run unconventional monetary policy shocks". The interpretation is quite simple, as advantages from TVP-VAR by allowing state-vectors to be time-variant via state-space model. Each impulse response function is derived from time to time one quarter by quarter. Therefore when we plot it all at one figure. It shows if impulse response to specific shock is actually changing over the periods. fig. 3b for instance, the thickness of time-varying impulse response is narrowed. This implies that the response of industrial production growth to BOJ discount rate is less volatile compared to response to short run interest rates in fig. 3a. This interpretation aid goes similarly to impulse response of inflation. figs. 3c and 3d shows impulse responses of inflation to nominal short-run interest rate shocks and discount rate shock. A rise in inflation after monetary tightening using VAR model is well known as the price puzzle, see Sims (1992) Response to short-run shocks are more bulky. Despite what mention above, the shape of both response to shocks are pretty identical.

Another point that is worth to discuss is according to fig. 5 Consumer Price Index responses significantly through the monetary contraction during the period between 1990 through late 1992. After that there is a huge expansionary monetary policy which actually drives Japanese inflation to increase steadily. Our statistical evidence from Historical Shock Decomposition functions are capable of tracking those volatility and this is quite reasonable and accurate. Past literature as in Kasa and Popper (1997) happens to find the identical results. Furthermore, as oil shocks occured during the period of 1981 through 1985, the Japanese consumer price index is extremely volatile and responsive to the shocks of BOJ discount rate.

[3] in our work we choose Discount Rate as the main monetary policy.

Table 1: Sign Restriction to Short-Run Interest Rate Shocks.

Responding Variable	Sign Restriction
Output Percent Growth	+
Consumer Price Index	+
Treasury Bill	-
Discount Rate	+

Table 2: Sign Restriction to Long-Run Interest Rate Shocks.

Responding Variable	Sign Restriction
Output Percent Growth	+
Consumer Price Index	+
Treasury Bill	+
Discount Rate	-

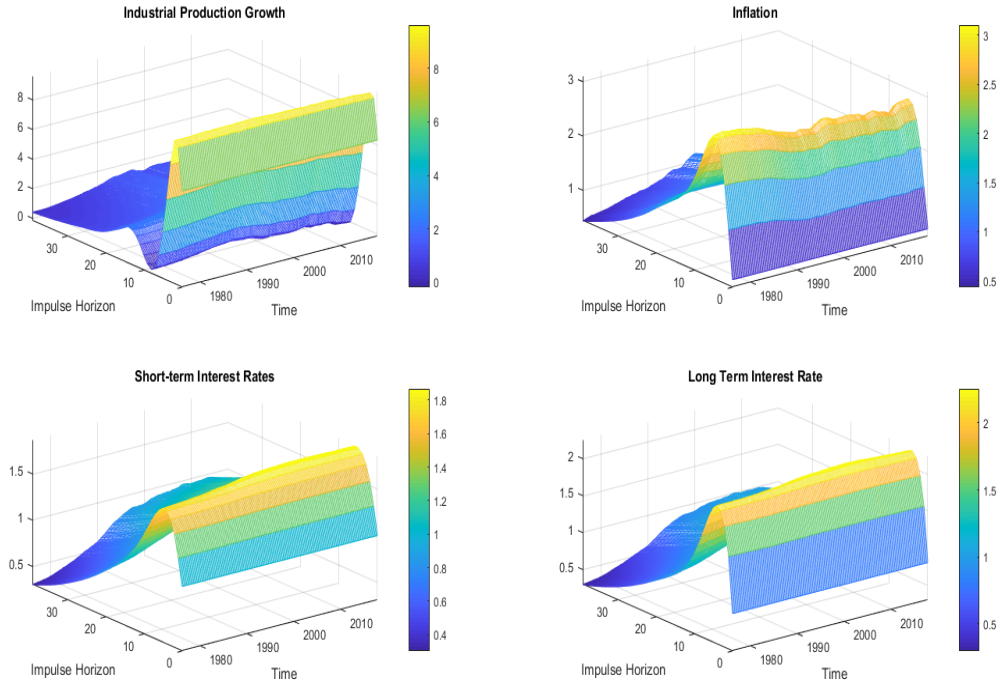

Figure 1: Impulse Response of Four Selected Macroeconomic Variables to Structural Shocks of Short-term Interest Rate (Treasury Bill Rates)

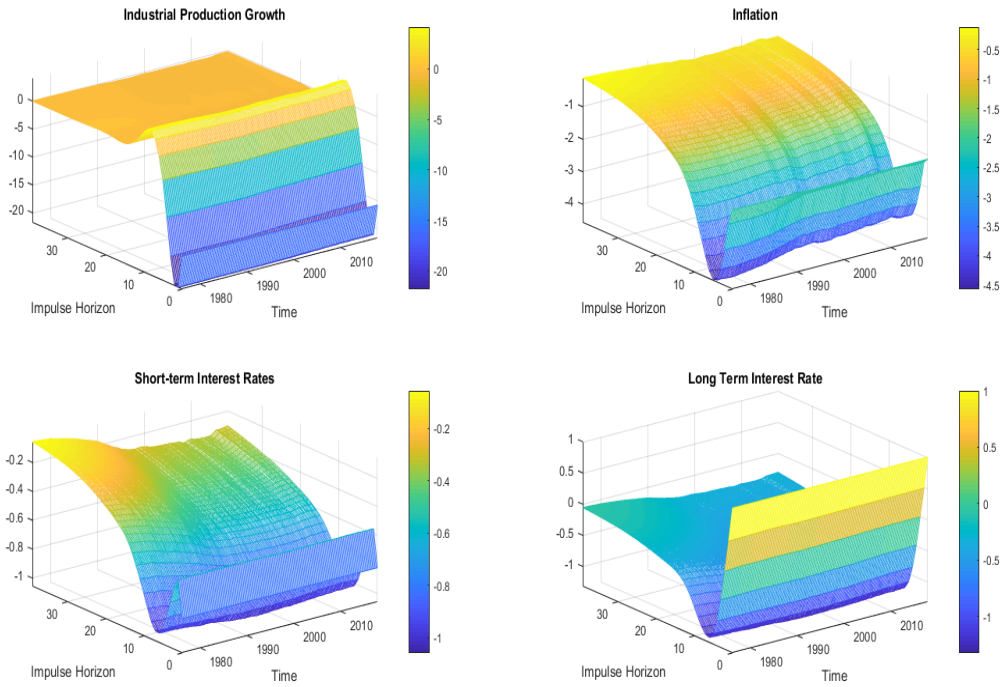

Figure 2: Impulse Response of Four Selected Macroeconomic Variables to Structural Shocks of Long-term Interest Rate (Discount Rates)

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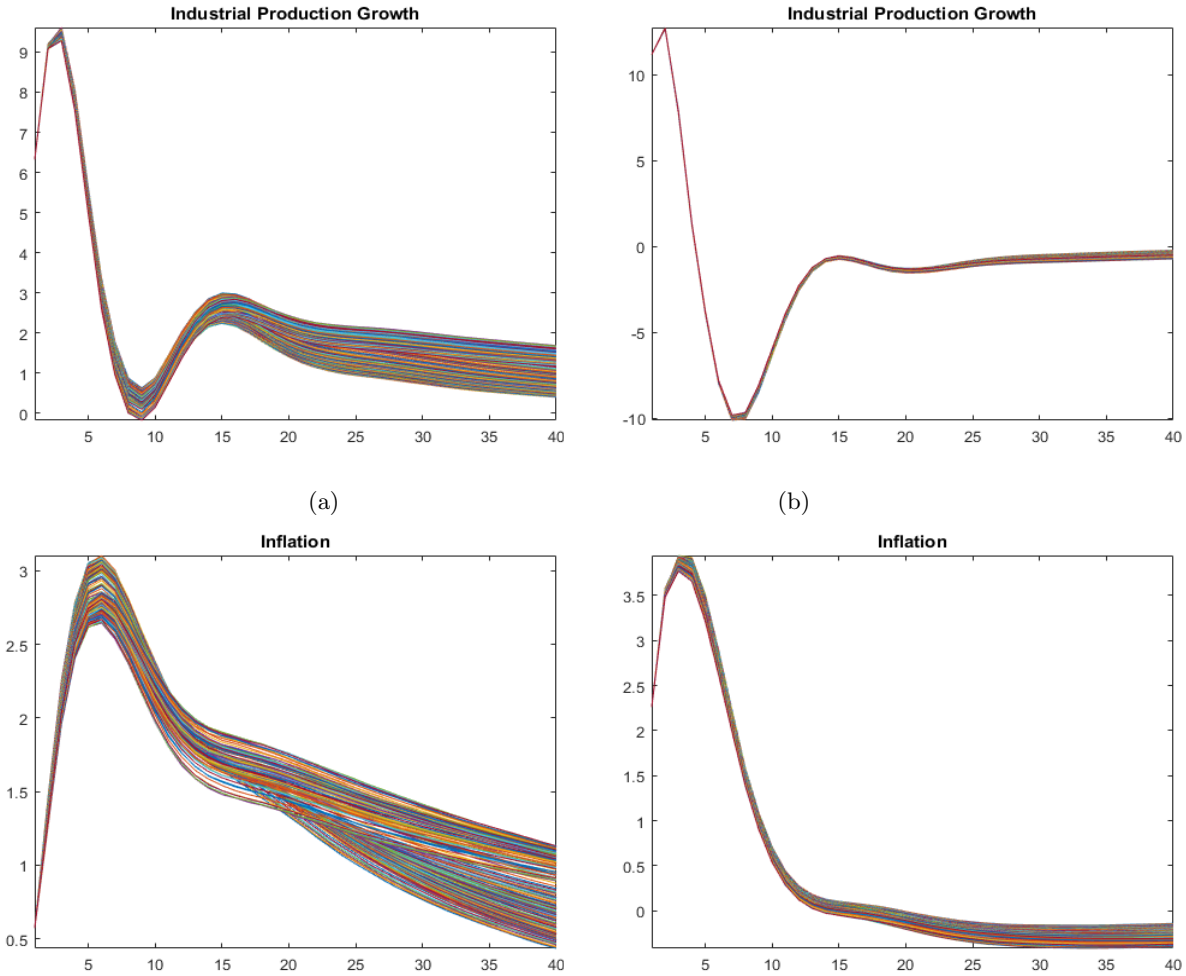
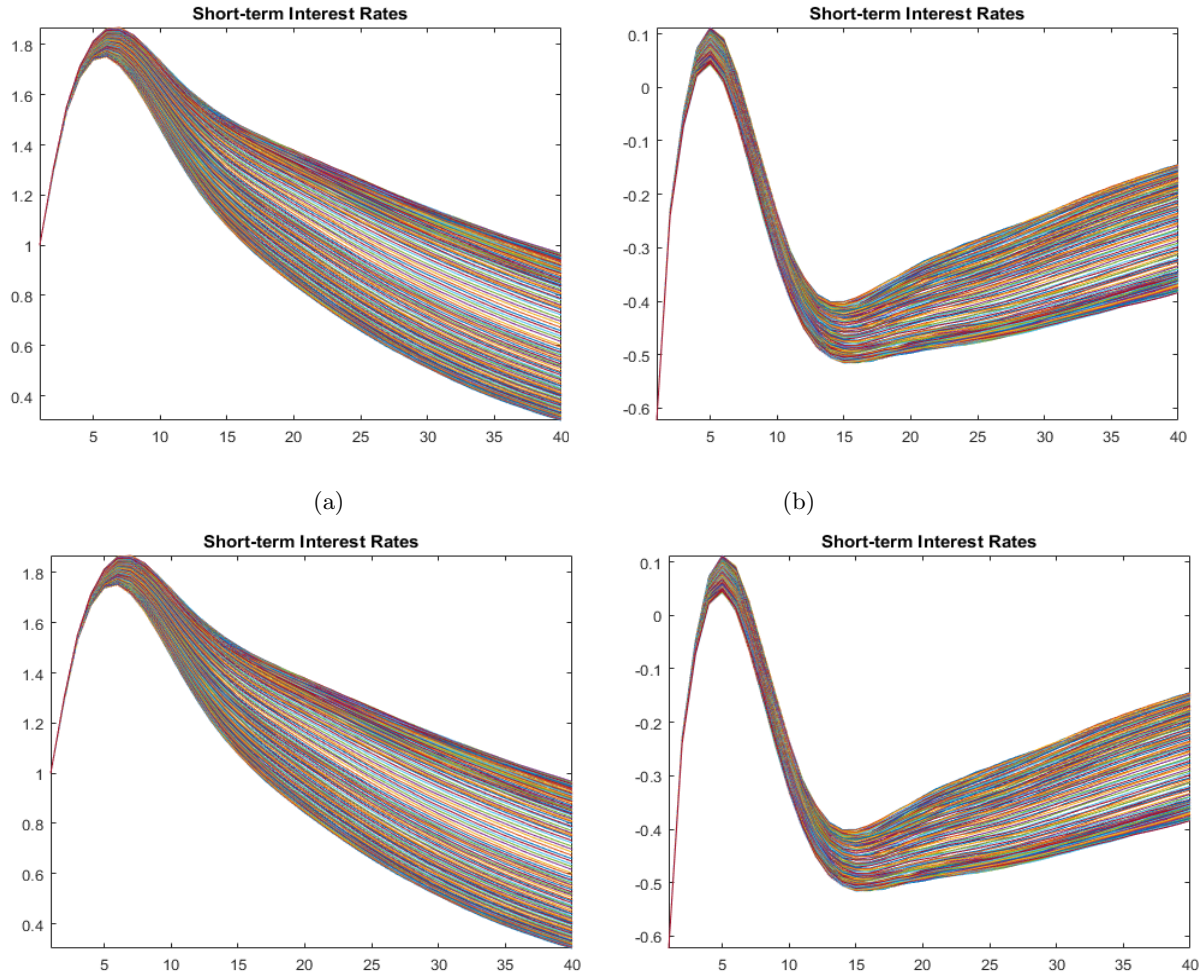


(a) (b)

(c) (d)

Figure 3: Note: Time-varying Impulse Response of Industrial Production Growth and Inflation to Short-term interest rate shocks figs. 3a and 3c and Long-term interest rate shocks figs. 3b and 3d, respectively.



(a) (b)

(c) (d)

Figure 4: Note: Time-varying Impulse Response of Treasury-Bill and Discount Rate to Short-term interest rate shocks figs. 4a and 4c and Long-term interest rate shocks figs. 4b and 4d, respectively.

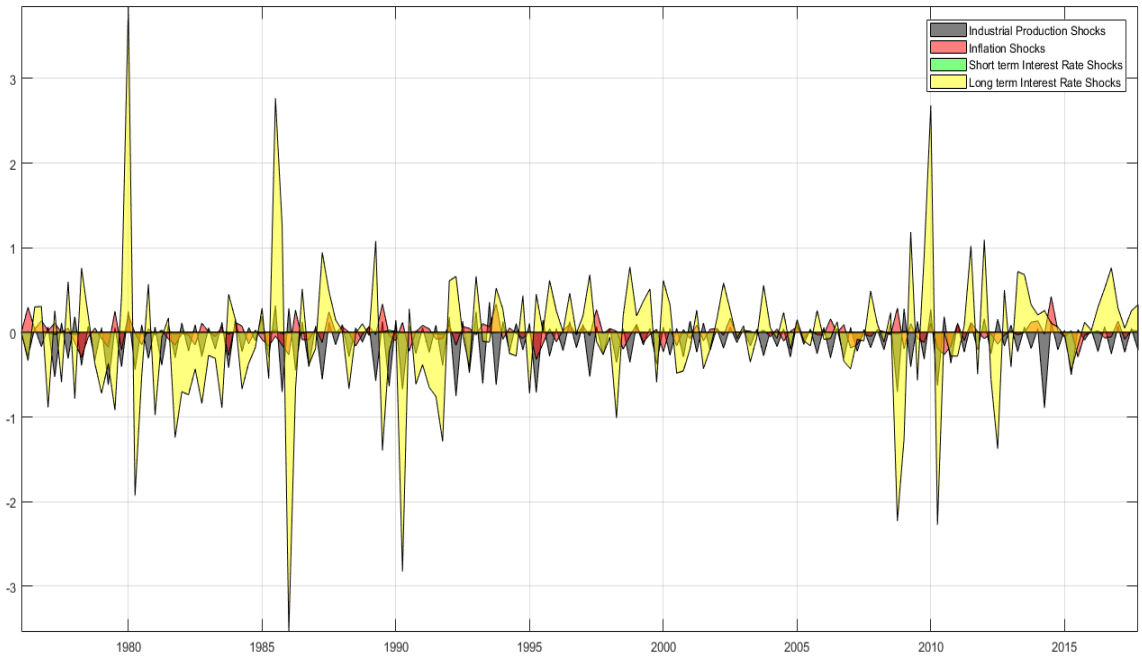


Figure 5: Note: Japanese Inflation: Historical shock decomposition obtained from eq. (15)

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